Syllabus : Chapter 2

- Conservative nature of Electrostatic Field
- Electrostatic Potential.
- Laplace's and Poisson equations.
- The Uniqueness Theorem.
- Potential and Electric Field of a dipole.
- Force and Torque on a dipole.

Conservative force

Let us consider a force F such that

$$\oint \vec{F} \cdot \vec{dl} = W = 0$$

Line integral is independent of the path. Then the force F is called conservaive force.

Electric Potential

What is electric Potential?

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}.$$

Path Independent..

$$\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E}\cdot d\mathbf{l} = \frac{1}{4\pi\epsilon_0}\int_{\mathbf{a}}^{\mathbf{b}} \frac{q}{r^2}dr = \frac{-1}{4\pi\epsilon_0}\frac{q}{r}\bigg|_{r_a}^{r_b} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right),$$

$$\oint \mathbf{E}\cdot d\mathbf{l} = 0,$$

Using Stokes Law

$$\nabla\times\mathbf{E} = 0.$$

$$\mathbf{E} = -\nabla V.$$

V is Potential

Electric Potential

What is electric Potential?

$$V(\mathbf{r}) \equiv -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}.$$

Here $\mathcal{O}$ is some standard reference point on which we have agreed beforehand;

Example from Griffiths:

Find the potential inside and outside a spherical shell of radius R (Fig. 2.31), which carries a uniform surface charge. Set the reference point at infinity.

Electric Potential

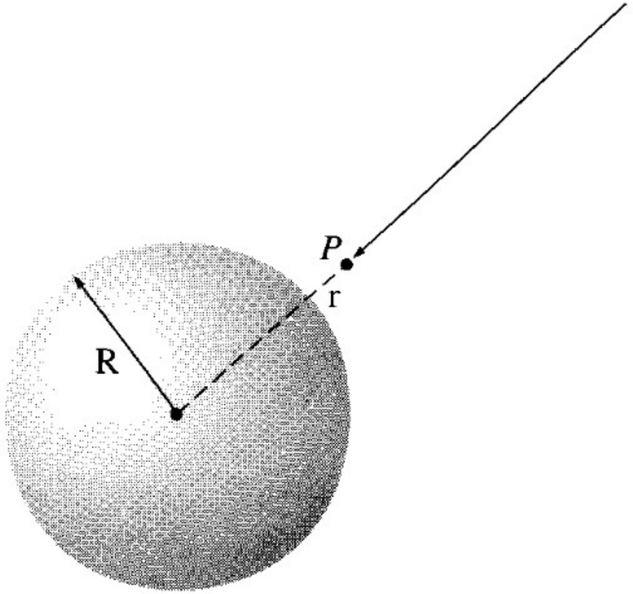
Find the potential inside and outside a spherical shell of radius R (Fig. 2.31), which carries a uniform surface charge. Set the reference point at infinity.

For r > R

$$V(r) = -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l} = \frac{-1}{4\pi\epsilon_0}\int_{\infty}^{r} \frac{q}{r'^2}\, dr' = \frac{1}{4\pi\epsilon_0}\frac{q}{r'}\bigg|_{\infty}^{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}.$$

For r < R

$$V(r) = \frac{-1}{4\pi\epsilon_0}\int_{\infty}^{R} \frac{q}{r'^2}\, dr' - \int_{R}^{r} (0)\, dr' = \frac{1}{4\pi\epsilon_0}\frac{q}{r'}\bigg|_{\infty}^{R} + 0 = \frac{1}{4\pi\epsilon_0}\frac{q}{R}.$$



Laplace's and Poisson's equations.

Relation between field and potential: $\mathbf{E} = -\nabla V.$

To know a vector completely you need to know divergence and curl..

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad \text{and} \quad \nabla \times \mathbf{E} = 0,$$

Using

$$\nabla \cdot \mathbf{E} = \nabla \cdot (-\nabla V) = -\nabla^2 V,$$

Poisson Equation: $\nabla^2 V = -\frac{\rho}{\epsilon_0}.$

Laplace's and Poisson's equations.

Poisson Equation: $$\nabla^2 V = -\frac{\rho}{\epsilon_0}.$$

In the charge free (charge density $\rho = 0$) space we get Laplace's equation:

Laplace's equation,

$$\nabla^2 V = 0.$$

The Uniqueness Theorem.

It is all about the solution of the Laplace equation. The potential V has no local max or min. The extrema value of V comes at the end points (for one dimension) or at boundary (for 2 or 3D).

The theorem has two parts:

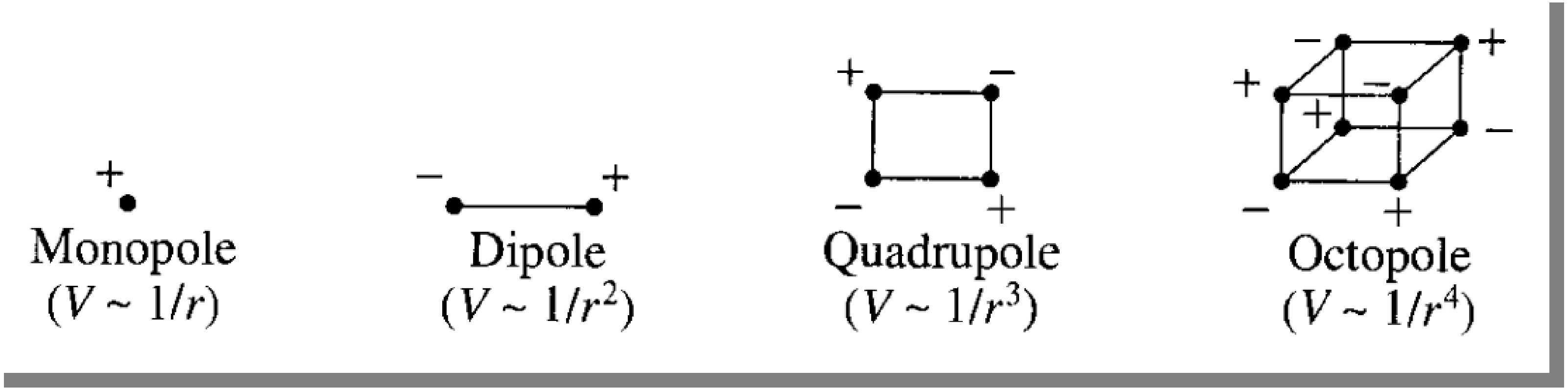
1. The potential function V(x,y,z) is uniquely determined in a volume if the charge density ρ is specified and the value of V(x,y,z) on all boundary is also specified.

2. In a volume surrounded by conductors and containing a specified charge density ρ, the electric field is uniquely determined if the total charge on each conductor is given.

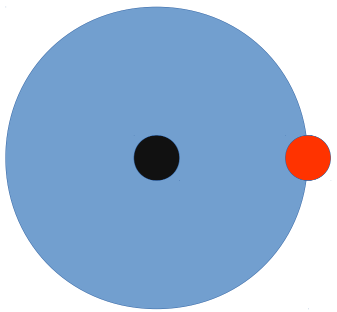
Electric Dipole

What happens to the potential if the system has total charge zero?

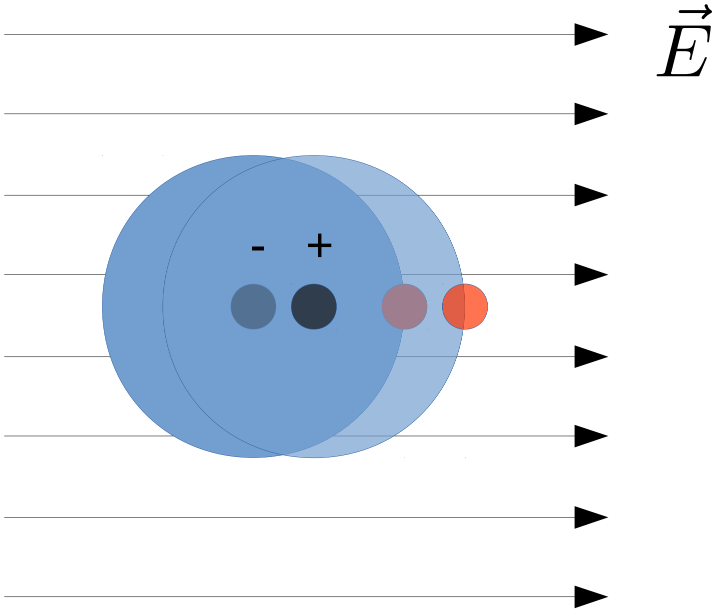
Not easy to answer so quickly..



Electric Dipole

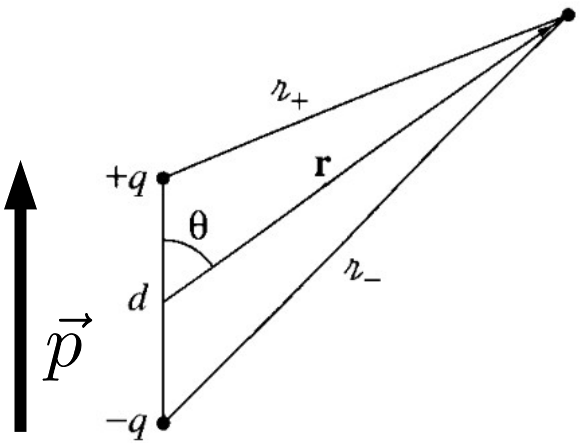


Neutral Atom



Dipole moment $\boldsymbol{p}$ is created

Electric Dipole



$$V_{dip}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}.$$

Potential at a distance r (r>> d)

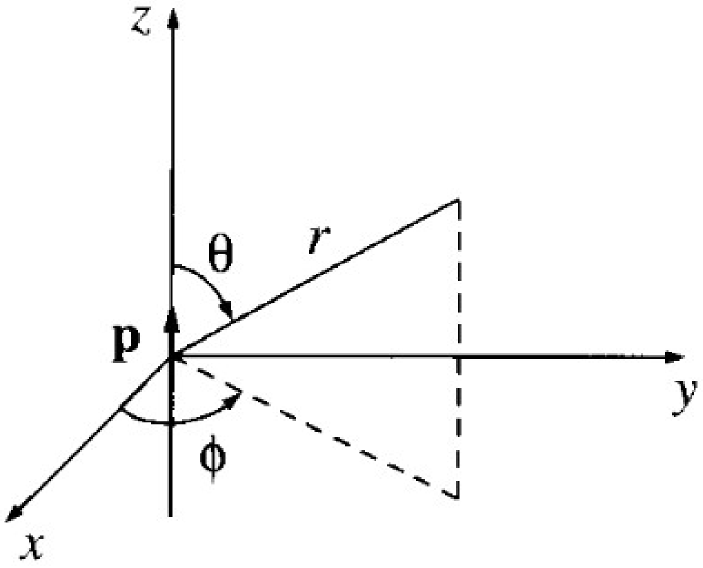
$$V(\mathbf{r}) \cong \frac{1}{4\pi\epsilon_0}\frac{qd\cos\theta}{r^2}.$$

$$E_r = -\frac{\partial V}{\partial r} = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3},$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{p\sin\theta}{4\pi\epsilon_0 r^3},$$

$$E_\phi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} = 0.$$

Electric Dipole



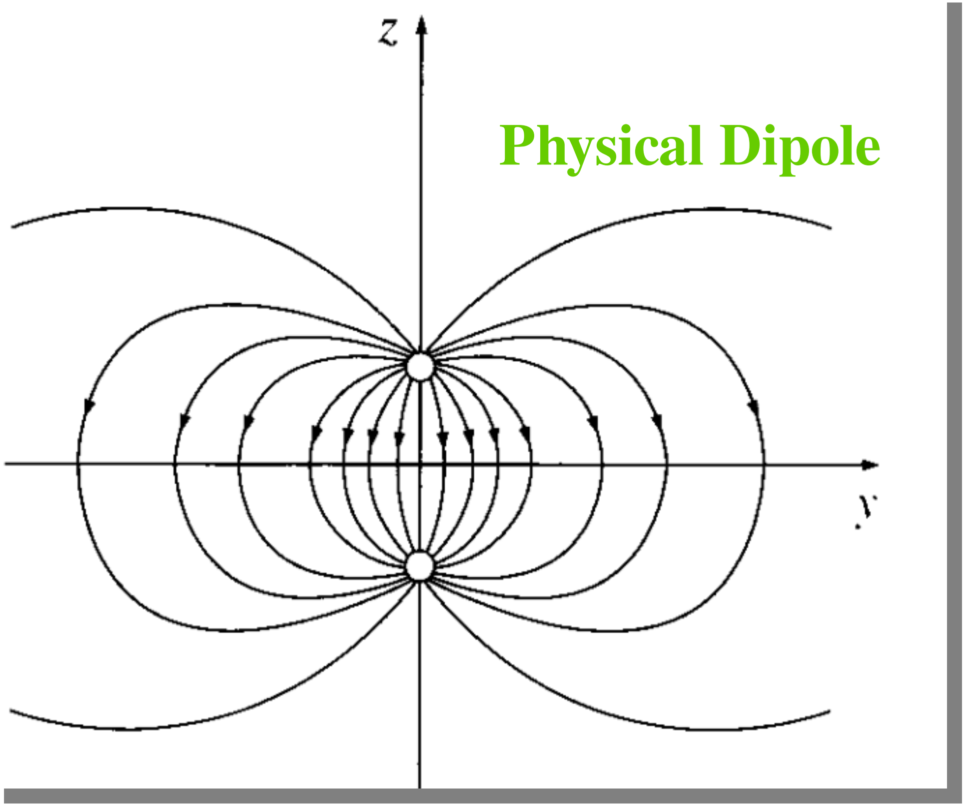
Potential at a distance r (r>> d)

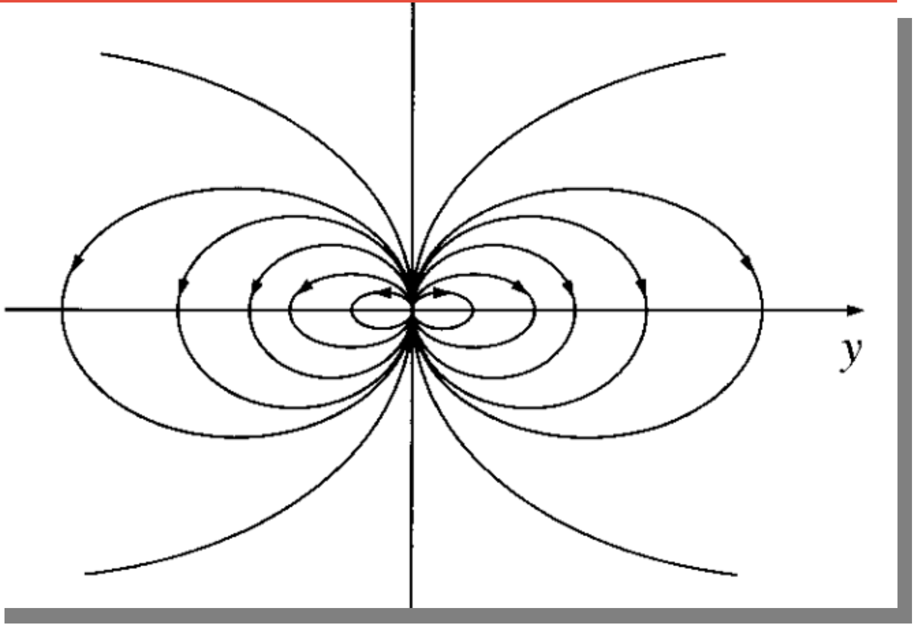
$$V(\mathbf{r}) \cong \frac{1}{4\pi\epsilon_0}\frac{qd\cos\theta}{r^2}.$$

$$V_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}.$$

$$\mathbf{E}_{\text{dip}}(r,\theta) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}).$$

Electric Dipole

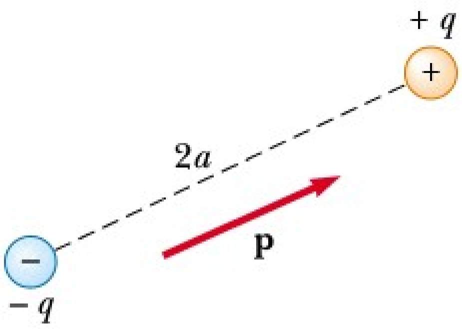


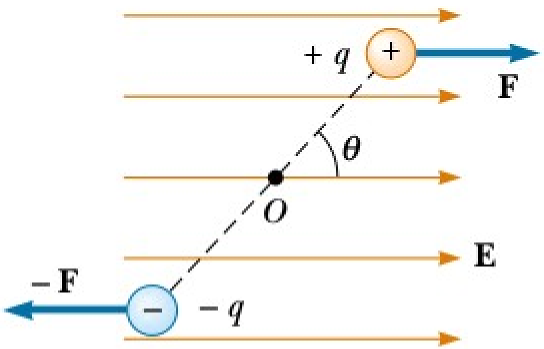


For a Pure dipole

$$\mathbf{E}_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}].$$

Dipole placed in an Electric Field





Amount of Torque:

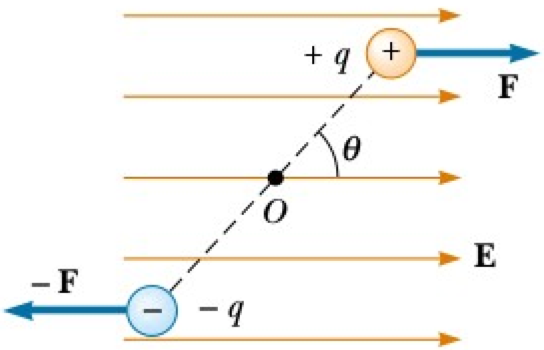
$$\tau = 2Fa \sin\,\theta$$

$$= 2aqE \sin\,\theta = pE \sin\,\theta$$

Force and Torque on a dipole

Amount of Torque:

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$



Potential Energy stored in the Dipole is

$$U = -\mathbf{p} \cdot \mathbf{E}$$